Precalc Warm Up # 9-4

1. Find all solutions on $[0, 2\pi)$ $\qquad$ $-6\tan 4x = 2\sqrt{3}$

2. Rewrite using the sum formulas:

a. sin 2u $\qquad$ b. cos 2u $\qquad$ c. tan 2u

Rewrite using the sum and difference formulas:

a. sin 2u

$\sin(u+u)$

$\sin u\cos u + \cos u \sin u$

$\boxed{2\sin u\cos u}$

b. cos 2u

$\cos(u+u)$

$\cos u\cos u - \sin u \sin u$

$\boxed{\cos^2 u - \sin^2 u}$

-or-

$1-\sin^2 u - \sin^2 u$

$\boxed{1-2\sin^2 u}$

or

$\cos^2 u - (1-\cos^2 u)$

$\boxed{2\cos^2 u - 1}$

c. tan 2u

$\tan(u+u)$

$\dfrac{\tan u + \tan u}{1-\tan u\tan u}$

$\boxed{\dfrac{2\tan u}{1-\tan^2 u}}$

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Verify:

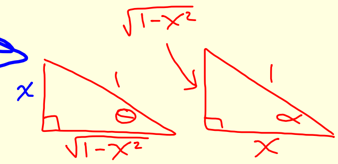
35. $\cos(\pi - \theta) + \sin\left(\frac{\pi}{2} + \theta\right) = 0$

37. $\tan(\pi + \theta) = \tan\theta$

55. $\sin(\arcsin x + \arccos x)$

$\theta \qquad \alpha$

$\sin(\theta + \alpha)$

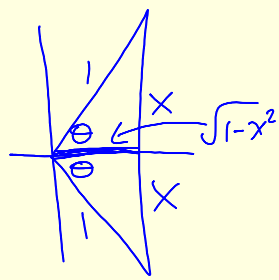


$\sin\theta\cos\alpha + \cos\theta\sin\alpha$

$x \cdot x + (\sqrt{1-x^2})(\sqrt{1-x^2})$

$x^2 + 1 - x^2$

$\boxed{1}$



In Exercises 57–62, find all solutions in the interval $[0, 2\pi)$.

57. $\sin\left(x + \frac{\pi}{3}\right) + \sin\left(x - \frac{\pi}{3}\right) = 1$

59. $\cos\left(x + \frac{\pi}{4}\right) + \cos\left(x - \frac{\pi}{4}\right) = 1$

$\cos x\cos\frac{\pi}{4} - \cancel{\sin x\sin\frac{\pi}{4}} + \cos x\cos\frac{\pi}{4} + \cancel{\sin x\sin\frac{\pi}{4}}$

$2\cos x\cos\frac{\pi}{4} = 1$

$\cancel{2}(\cos x)\frac{\sqrt{2}}{\cancel{2}} = 1$

$\cos x = \frac{\sqrt{2}}{2}$ I & IV

$x = \frac{\pi}{4}, \frac{7\pi}{4}$

57)

$\sin x\cos\frac{\pi}{3} + \cancel{\cos x\sin\frac{\pi}{3}} + \sin x\cos\frac{\pi}{3} - \cancel{\cos x\sin\frac{\pi}{3}} = 1$

$2\sin x\cos\frac{\pi}{3} = 1$

$\cancel{2}(\sin x)\left(\frac{1}{\cancel{2}}\right) = 1$

$x = \sin^{-1}(1)$

$x =$

In Exercises 57–62, find all solutions in the interval $[0, 2\pi)$.

61. $\tan(x + \pi) + 2\sin(x + \pi) = 0$

$$\frac{\tan x + \tan\pi}{1 - \tan x \tan\pi} + 2(\sin x\cos\pi + \cos x\sin\pi) = 0$$

$$\frac{\tan x}{1 - 0} + 2((\sin x)(-1) + (\cos x)(0)) = 0$$

$$\tan x - 2\sin x = 0$$

$$\frac{\sin x}{\cos x} - 2\sin x = 0$$

$$\sin x\left(\frac{1}{\cos x} - 2\right) = 0$$

$\sin x = 0$ $\qquad$ $\frac{1}{\cos x} = \frac{2}{1}$

$x = 0, \pi$ $\qquad$ $\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

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In Exercises 7–20, find all solutions of the equation. (Do not use a calculator.)

9. $\sqrt{3}\csc x - 2 = 0$

$$\csc x = \frac{2}{\sqrt{3}}$$

$$\sin x = \frac{\sqrt{3}}{2}$$

$$x =$$

13. $3\sec^2 x - 4 = 0$

17. $\sin x(\sin x + 1) = 0$

In Exercises 21–40, find all solutions in the interval $[0, 2\pi)$. (Do not use a calculator.)

$\boxed{21.}$ $\sec x \csc x - 2 \csc x = 0$

$\csc x(\sec x - 2) = 0$

$\cancel{\csc x = 0}$

$\sec x = 2$

$\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

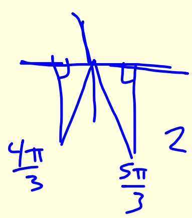
$\boxed{25.}$ $\cos^3 x = \cos x$

$\boxed{29.}$ $2 \sec^2 x + \tan^2 x - 3 = 0$

$2(1 + \tan^2 x) + \tan^2 x - 3 = 0$

$\boxed{33.}$ $\sin 2x = -\frac{\sqrt{3}}{2}$

$2x = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$



$\leftarrow$ $2x = \frac{4\pi}{3}, \frac{5\pi}{3}$

$2x = \frac{4\pi}{3} + 2\pi n$ | $2x = \frac{5\pi}{3} + 2\pi n$

$x = \frac{2\pi}{3} + \pi n$ | $x = \frac{5\pi}{6} + \pi n$

Now list on $[0, 2\pi)$

37. $\frac{1 + \cos x}{1 - \cos x} = 0$

$1 + \cos x = 0$

$\cos x = -1$

Solve: $-2\sin 2x = 1$ using two different methods. $[0,2\pi)$

method 1: Find the general solution and use it to generate all solutions in the interval.

$\sin 2x = -1/2$

$2x = 7\pi/6 + 2\pi n$, and $2x = 11\pi/6 + 2\pi n$

$x = 7\pi/12 + \pi n$, and $x = 11\pi/12 + \pi n$

$\boxed{7\pi/12, \quad 11\pi/12, \quad 19\pi/12, \quad 23\pi/12}$

method 2: use the multiple angle identity

$-2\sin 2x = 1$

$\sin 2x = -1/2$

$2\sin x\cos x = -1/2$ I don't like where this is going.....

$\sin x\cos x = -1/4$

$\sin^2 x\cos^2 x = 1/16$

$\sin^2 x(1-\sin^2 x) = 1/16$

$\sin^2 x - \sin^4 x - 1/16 = 0$ I still don't like where this is going

$\sin^4 x - \sin^2 x + 1/16 = 0$ It looks like it will factor, but alas, it won't!

let $a = \sin^2 x$ thus $a^2 - a + 1/16 = 0$

$a \approx .933$ or $.067$

so $\sin^2 x \approx .933$ or $\sin^2 x \approx .067$

$\sin x \approx .966$, $\sin x \approx -.966$, $\sin x \approx .259$, $\sin x \approx -.259$

$x \approx 1.31, x \approx 1.83$, $x \approx -1.31\ (4.97), x \approx 4.45$ $x \approx .26, x \approx 2.88$, $x \approx -.26(6.02), x \approx 3.4$

since we squared, some of these are extraneous.

The final answers are 1.83, 2.88, 4.97, 6.02 which is what we got before.

Obviously, this was the HARD WAY!

Some problems are well suited to using the double angle formulas. Solve:

$$2\cos x + \sin 2x = 0$$
$$2\cos x + 2\sin x\cos x = 0$$
$$2\cos x\,(1 + \sin x) = 0$$
$$2\cos x = 0 \quad \text{or} \quad 1 + \sin x = 0$$
$$\cos x = 0 \qquad\qquad \sin x = -1$$

$$\boxed{\pi/2,\ \ 3\pi/2}$$

Solve: $\cos 2x = 2\sin^2 x$ on $[0,2\pi)$

$$1 - 2\sin^2 x = 2\sin^2 x$$
$$1 = 4\sin^2 x$$
$$1/4 = \sin^2 x$$
$$\pm 1/2 = \sin x$$

$$x = \boxed{\pi/6,\ \ 5\pi/6,\ \ 7\pi/6,\ \ 11\pi/6}$$

Use the double angle formulas in reverse order.

$$4 \sin x \cos x = 1$$

Because of the 1, the factors here don't help us. But if you rewrite it using the double angle formula you get:

$$2(2\sin x \cos x) = 1$$
$$2 \sin 2x = 1$$
$$\sin 2x = 1/2$$

Easy from there!

It is sometimes helpful to reduce the power in an expression, if we don't mind multiple angles

Using the double angle formula for cosine,

we have $\cos 2u = 1 - 2 \sin^2 u$

$2\sin^2 u = 1 - \cos 2u$

$$\boxed{\sin^2 u = \frac{1 - \cos 2u}{2}}$$

likewise, $\cos 2u = 2 \cos^2 u - 1$

$2 \cos^2 u = 1 + \cos 2u$

$$\boxed{\cos^2 u = \frac{1 + \cos 2u}{2}}$$

Since $\tan^2 u = \dfrac{\sin^2 u}{\cos^2 u}$

$$\boxed{\tan^2 u = \frac{1 - \cos 2u}{1 + \cos 2u}}$$

These are the Power Reducing Formulas

Using the Power Reducing formulas to solve:

$$\cos^2 \underbrace{2x}_{u} - \sin^2 \underbrace{2x}_{u} = -1/2$$

$$\frac{1+\cos 2(2x)}{2} - \frac{1-\cos 2(2x)}{2} = -\frac{1}{2}$$

$$1 + \cos 4x - 1 + \cos 4x = -1$$

$$2\cos 4x = -1$$

$$\cos 4x = -\frac{1}{2}$$

$$4x = \cos^{-1}\left(-\frac{1}{2}\right)$$

$$4x = \frac{2\pi}{3} + 2\pi n \qquad 4x = \frac{4\pi}{3} + 2\pi n$$

$$x = \frac{\pi}{6} + \frac{\pi}{2}n \qquad x = \frac{\pi}{3} + \frac{\pi}{2}n$$

Use the Pythogorean Identities to solve.

$$\cos^2 2x - \sin^2 2x = -1/2$$

$$\cos^2 2x - (1 - \cos^2 2x) = -\frac{1}{2}$$

$$2\cos^2 2x - 1 = -\frac{1}{2}$$

$$\sqrt{\cos^2 2x} = \sqrt{\frac{1}{4}}$$

$$2x = \cos^{-1}\left(\pm\frac{1}{2}\right)$$

$$2x = \frac{\pi}{3} + \pi n \qquad 2x = \frac{2\pi}{3} + \pi n$$

$$x = \frac{\pi}{6} + \frac{\pi}{2}n \qquad x = \frac{\pi}{3} + \frac{\pi}{2}n$$

Or the Double Angle Formulas:

$\cos^2 2x - \sin^2 2x = -1/2$

Let $u = 2x$

$\cos^2 u - \sin^2 u$

$\cos 2u$

$\cos 2(2x) = -\frac{1}{2}$

$\cos 4x = -\frac{1}{2}$

$4x = \cos^{-1}\left(-\frac{1}{2}\right)$

$4x = \frac{2\pi}{3} + 2\pi n$ $\quad$ $4x = \frac{4\pi}{3} + 2\pi n$

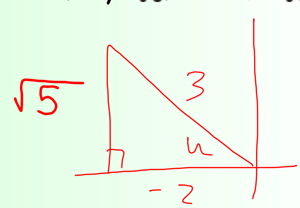
$x = \frac{\pi}{6} + \frac{\pi}{2}n$ $\quad$ $x = \frac{\pi}{3} + \frac{\pi}{2}n$

Find the exact value of sin 2u if $\cos u = -2/3$, $\pi/2 < u < \pi$

$\sin 2u = 2\sin u \cos u$

$= 2\left(\frac{\sqrt{5}}{3}\right)\left(-\frac{2}{3}\right)$

$= -\frac{4\sqrt{5}}{9}$



Rewrite without multiple angles

a. $\sin 4x$

$\sin 2(2x)$

$2\sin 2x \cos 2x$

$2(2\sin x\cos x)(1 - 2\sin^2 x)$

$4\sin x\cos x\,(1 - 2\sin^2 x)$

b. $\sin 3x$

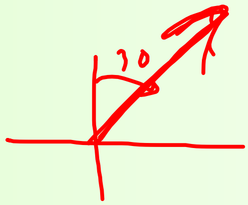
$\sin(x + 2x)$

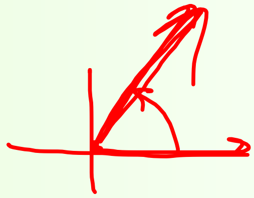
$\sin x\cos 2x + \cos x\sin 2x$

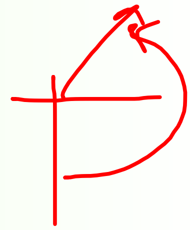
$\sin x\,(1 - 2\sin^2 x) + \cos x(2\sin x\cos x)$

$\sin x - 2\sin^3 x + 2\sin x\cos^2 x$

N 30°E is the same as E 60°N, or even S 150° E !

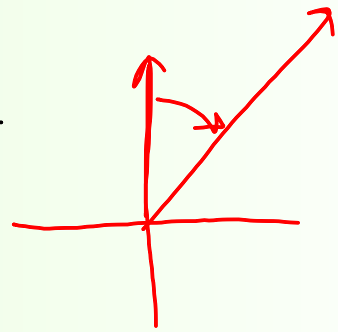






To standardize this, we have what's called TRUE bearing. True bearing is always measured clockwise from North.

N 30° E = 30° T



Give the True Bearing for:

1. S 50°E

130° T

2. N 42°W

318° T

HW: p. 440 #1-11 odd,

51, 53, 56, 57, 73-75

Quiz 6.1-6.3 tomorrow,
without 6.4-6.5 formulas